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ON SUBSONIC LAMINAR SEPARATION FROM THE DISCONTINUITY EDGE OF A PROFILE*

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Asymptotic methods are used to study a steady subsonic flow of a perfect gas past a convex angle at high Reynolds' numbers. The solution /1/ describing a potential flow past a corner with a free streamline is taken as the limit solution. The pressure gradient in this solution tends to infinity on approaching the corner point from the direction of incoming flow. Its effect on the boundary layer is to form a domain of free interaction, first studied in /2—4/. The Navier—Stokes equations were solved outside the domain of free interaction in the complete heighborhood of the corner in /1/. The flow in the domain of free interaction was studied under the assumption that the surface of the corner is thermally insulated. The problem describing this flow in the first approximation is reduced by means of affine transformations to a problem of laminar separation of an incompressible fluid /5/. This makes it possible to establish a similarity law for subsonic gas flows in the neighborhood of a corner point.

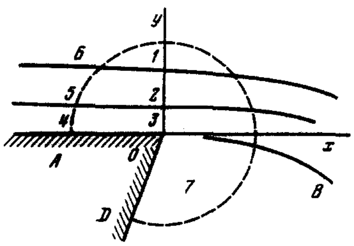

Fig.1

We consider an irrotational flow past a convex corner AOD of a perfect gas with a free streamline OB emerging from the apex of the corner (see Fig.1). The velocity of the gas is assumed subsonic. We introduce the Cartesian x, y -coordinate system in which the negative semiaxis x coincides with AO. We denote by v_x and v_y the projections of the velocity vector on the x and y axis respectively, ρ is density, p is pressure, T is temperature, M is the Mach No., μ in the coefficient of viscosity, q_0 is the velocity at the corner point of the external potential flow, and L is its characteristic dimension. Below we assume all equations to be dimensionless. The values of the flow parameters at the corner point are taken as characteristics, and denoted by zero subscript.

The solution constructed in /1/ which corresponds to a flow past a corner with a free streamline, holds in the range $0 \leqslant M_0 \leqslant 1$. When $M_0 < 1$, the solution in the neighborhood of the corner point with dimensions $l < \varepsilon^3 d_{11}/6$, $\varepsilon = 1 - M_0^2$ (region 6) can be written in the first approximation as

$$\varphi = \frac{2^{3/2}}{3\sqrt{d_{11}\varepsilon}} \mathrm{Re}\,[\mathrm{i}\,(x + \mathrm{i}\sqrt{\varepsilon}\,y)^{3/2}] + \ldots \tag{1}$$

Here d_{11} is a constant determined by the global solution and φ is the perturbation potential. The corresponding favorable pressure gradient tends as $x \to 0$ $(x < 0)$, with fixed ε and d_{11}, to infinity as $(-x)^{-1/2}$. It interaction with the boundary layer results in formation of a domain of free interactions. Since the thickness of the filaments in narrow region adjacent to the wall changes drastically, it follows that the pressure in this region is induced by the boundary layer itself. We assume that the characteristic dimension of the domain of free interaction is $\Delta x \ll l$.

Let us denote the velocity and density of the boundary layer at the coordinate origin by $U(Y)$, $R(Y)$, $y = \mathrm{Re}^{-1/2} Y$. Analysis of the flow in the attached region has shown /1/ that $U(Y) \to 1$, $R(Y) \to 1$, $Y \to \infty$

$$U(Y) = {}^5/_3 b_0 Y^{2/3} + \ldots, \quad R(Y) = R(0) + c_0 Y^{4/3} + \ldots, \quad Y \to 0$$

We divide the field of flow in the domain of free interaction into three regions (see Fig.1). In the upper region *1* the viscosity and heat conductivity can both be neglected and the flow in it is not potential. In the middle region *2* the dissipative factors have no significant influence, but the velocity field is rotational. In the narrow region *3* adjacent to the wall the flow is governed mainly by the viscous stresses. The heat conductivity exerts a second order influence, since the gas is practically incompressible at low velocities and within the temperature regimes in question.

We seek, in the basic part of the boundary layer (region 2) a solution of the Navier—Stokes equations in the form of series

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$$V_x = U(Y_2) + \mathrm{Re}^{-1/9}u_0(X, Y_2) + \mathrm{Re}^{-2/9}u_1(X, Y_2) + \ldots$$
$$\rho = R(Y_2) + \mathrm{Re}^{-1/9}\rho_0(X, Y_2) + \mathrm{Re}^{-2/9}\rho_1(X, Y_2) + \ldots$$
$$V_y = \mathrm{Re}^{1/9}V_0(X, Y_2) + \mathrm{Re}^{2/9}V_1(X, Y_2) + \ldots, \ v_y = \mathrm{Re}^{1/2}V_y$$
$$p = 1 + \gamma M_0^2\mathrm{Re}^{-2/9}p_0(X) + \ldots, \ x = \mathrm{Re}^{-4/9}X, \ Y_2 = Y = \mathrm{Re}^{1/2}y \tag{2}$$

where $\mathrm{Re} = \rho_0 q_0 L/\mu_0$ (γ is the ratio of specific heats. We obtain u_i, V_i, ρ_i $(i = 0, 1)$ from a system of ordinary differential equations. Integrating it we obtain ($A_0(X)$ is an arbitrary function): for $Y_2 \to 0$

$$v_x = \frac{5}{3} b_0 Y_2^{2/3} + \frac{10}{9} b_0 A_0(X)\,\mathrm{Re}^{-1/9} Y_2^{-1/3} + \frac{3}{5}\frac{p_0(X)}{b_0 R(0)}\mathrm{Re}^{-2/9} Y_2^{-2/3} + \ldots$$
$$\rho = R(0) + \mathrm{Re}^{-2/9}\left[c_0 Y_2^{4/3} + \frac{4}{3} c_0 A_0(X) Y_2^{1/3} + \left(\frac{36}{25}\frac{c_0}{R(0)} b_0^2 + M_0^2 R(0)\right) p_0(X)\right] + \ldots$$
$$V_y = -\frac{5}{3} b_0 \frac{dA_0}{dX}\mathrm{Re}^{5/18} Y_2^{2/3} + \frac{9}{5 b_0 R(0)}\mathrm{Re}^{0/9} Y_2^{1/3} + \ldots \tag{3}$$

and for $Y_2 \to \infty$

$$v_x = 1 - \mathrm{Re}^{-2/9}p_0(X) + \ldots, \ \rho = 1 + \mathrm{Re}^{-2/9}M_0^2 p_0(X) + \ldots$$
$$v_y = -\mathrm{Re}^{-2/9}\frac{dA_0}{dX} + \ldots, \quad p = 1 + \gamma M_0^2 p_0(X)\,\mathrm{Re}^{-2/9} + \ldots \tag{4}$$

In the lower region *3* adjacent to the wall the solution is sought in the form

$$v_x = \mathrm{Re}^{-1/9}u_0(X, Y_3) + \ldots, \ V_y = \mathrm{Re}^{1/9}V_0(X, Y_3) + \ldots$$
$$\rho = R(0) + \mathrm{Re}^{-1/9}\rho_0(X, Y_3) + \mathrm{Re}^{-2/9}\rho_1(X, Y_3) + \ldots$$
$$p = 1 + \gamma M_0^2\mathrm{Re}^{-2/9}p_0(X) + \ldots, \ x = \mathrm{Re}^{-4/9}X, \ Y_2 = \mathrm{Re}^{-1/9}Y_3 \tag{5}$$

Substituting (5) into the equations of continuity and impulse, we obtain the following system of Prandtl equations:

$$\frac{\partial u_0}{\partial X} + \frac{\partial V_0}{\partial Y_3} = 0, \quad R(0)\left(u_0\frac{\partial u_0}{\partial X} + V_0\frac{\partial u_0}{\partial Y_3}\right) = -\frac{dp_0}{dX} + \frac{C}{R(0)}\frac{\partial^2 u_0}{\partial Y_3^2} \tag{6}$$

in which the pressure is not given and must therefore be determined. The viscosity coefficient is proportional to the temperature $\mu = CT$. When $X \to -\infty$, the velocity u_0 must merge with the velocity of the attached flow in region *4* /1/

$$u_0 = 2^{3/4}R^{-1/2}(0)\,d_{11}^{-1/4}\varepsilon^{-1/4}(-X)^{1/4}\frac{d\Phi_0}{d\zeta} + \ldots, \quad X \to -\infty \tag{7}$$

Function Φ_0 satisfies the boundary value problem with unique solution /6/

$$\frac{5}{2}\Phi_0\frac{d^2\Phi_0}{d\zeta^2} - \left(\frac{d\Phi_0}{d\zeta}\right)^2 - \frac{d^3\Phi_0}{d\zeta^3} = 1$$
$$\Phi_0(0) = \Phi_0'(0) = 0, \ \Phi_0(\zeta) = B_0\zeta^{5/3} + B_1\zeta^{2/3} + \ldots, \ \zeta \to \infty$$
$$b_0 = 2^{1/3}d_{11}^{-1/3}\varepsilon^{-1/3}B_0 \tag{8}$$

The constants B_0 and B_1 were obtained by numerical methods in /5,7/. The exapnsions (3) yield, as $Y_3 \to \infty$,

$$u_0 = \frac{5}{3} b_0 Y_2^{2/3} + \ldots, \quad V_y/v_x = -dA_0/dX + \ldots \tag{9}$$

At the wall $Y_3 = 0, X < 0$ the velocity components satisfy the condition of adhesion $v_x = V = 0$.

We shall assume that the Prandtl number is equal to unity and the wall is thermally insulated, i.e. $\partial T/\partial Y_3 = 0$ when $Y_3 = 0, X < 0$. Then $\rho_0(X, Y_3) = 0$ and the following integral can be used in determining the density:

$$\frac{T}{\gamma - 1} + M_0^2\frac{v_x^2}{2} = \frac{1}{(\gamma - 1)R(0)}$$

Using this integral with the equations of continuity, we find

$$\rho_1 = \frac{\gamma - 1}{2}R^2(0)M_0^2u_0^2 + \gamma R(0)M_0^2p_0(X) \tag{10}$$

We seek the solutions in the upper region (1) in the form of expansions

$$v_x = 1 + \mathrm{Re}^{-2/8} u_0(X, Y_1) + \ldots, \quad v_y = \mathrm{Re}^{-3/8} v_0(X, Y_1) + \ldots \qquad (11)$$
$$\rho = 1 + \mathrm{Re}^{-2/8}\rho_0(X, Y_1) + \ldots, \quad p = 1 + \mathrm{Re}^{-2/8}\gamma M_0^2 P_0(X, Y_1) + \ldots$$
$$y = \mathrm{Re}^{-4/8} Y_1, \quad x = \mathrm{Re}^{-4/8} X$$

By introducing the function of potential of perturbed velocities $\varphi_0(X, Y_1)$, we can make the flow in region 1 governable by the equations

$$(1 - M_0^2)\frac{\partial^2\varphi_0}{\partial X^2} + \frac{\partial^2\varphi_0}{\partial Y_1} = 0, \quad \rho_0(X, Y_1) = M_0^2 P_0(X, Y_1) = -M_0^2\frac{\partial\varphi_0}{\partial X} \qquad (12)$$

The boundary conditions at $X < 0$ are obtained by matching the expansions (11) and (4). We have

$$\frac{\partial\varphi_0}{\partial Y_1} = -\frac{dA_0}{dX}, \quad \frac{\partial\varphi_0}{\partial X} = -p_0(X), \quad Y_1 = 0, \quad X \leqslant 0 \qquad (13)$$

Considering the flow in stagnation zone /1,5/ we obtain

$$\partial\varphi_0/\partial X = 0, \; X > 0, \; Y_1 = 0 \qquad (14)$$

When $X^2 + Y_1^2 \to \infty$, the solution of the first equation of (12) must be transformed into (1), i.e.

$$\varphi_0 = \frac{2^{3/2}}{3} d_{11}^{-1/2}\varepsilon^{+1/4}\,\mathrm{Re}\left[\mathrm{i}\left(\frac{X}{\sqrt{\varepsilon}} + \mathrm{i}Y_1\right)^{3/2}\right] + \ldots, \; Y_1 > 0 \qquad (15)$$

To find the flow parameters in the domain of free interaction, we must solve the boundary value problems (6),(7),(9) and (12)–(15), together with the adhesion condition, which are related to each other. Let us transform the variables and flow parameters in the regions *1* and *3*

$$X = 2^{-3/8}C^{4/8}R^{-2/8}(0)\, d_{11}^{5/8}\varepsilon^{1/8}\bar{X}, \; Y_3 = \beta\bar{Y}_3, \; A_0(X) = \beta\bar{A}_0(\bar{X}) \qquad (16)$$
$$u_0 = 2^{1/8}C^{1/8}R^{-3/8}(0)\, d_{11}^{-1/8}\varepsilon^{-2/8}\bar{u}_0, \quad V_0 = 2^{1/8}C^{1/8}R^{-1}(0)\, d_{11}^{-1/8}\varepsilon^{-1/8}\bar{V}_0$$
$$p_0 = \chi\bar{p}_0, \quad P_0 = \chi\bar{P}_0, \quad \varphi_0 = 2^{-1/8}C^{2/8}R^{-1}(0)\, d_{11}^{1/8}\varepsilon^{-1/8}\bar{\varphi}_0$$
$$Y_1 = 2^{-3/8}C^{4/8}R^{-2/8}(0)\, d_{11}^{3/8}\varepsilon^{-7/16}\bar{Y}_1, \quad \beta = 2^{-1/8}C^{2/8}R^{-1}(0)\, d_{11}^{1/8}\varepsilon^{1/8}$$
$$\chi = 2^{2/8}C^{2/8}R^{-1/8}(0) d_{11}^{-3/8}\varepsilon^{-4/8}$$

The transformations (16) convert the boundary value problems (12)–(15) and (6),(7),(9), together with the condition of adhesion, into canonical form. In region *3* we have

$$\frac{\partial\bar{u}_0}{\partial\bar{X}} + \frac{\partial\bar{V}_0}{\partial\bar{Y}_3} = 0, \quad \bar{u}_0\frac{\partial\bar{u}_0}{\partial\bar{X}} + \bar{V}_0\frac{\partial\bar{u}_0}{\partial\bar{Y}_3} = -\frac{\partial\bar{p}_0}{\partial\bar{X}} + \frac{\partial^2\bar{u}_0}{\partial\bar{Y}_3^2} \qquad (17)$$
$$\bar{u}_0 = \frac{5}{3}B_0\bar{Y}_3^{2/5} + \ldots, \quad \frac{\bar{u}_0}{\bar{V}_0} = -\frac{d\bar{A}_0}{d\bar{X}} + \ldots, \quad \bar{Y}_3 \to \infty, \quad \bar{X} \leqslant 0$$
$$\bar{u}_0 = \sqrt{2}(-\bar{X})^{1/4}\frac{d\Phi_0}{d\zeta} + \ldots, \quad \zeta = 2^{-1/4}\bar{Y}_3/(-\bar{X})^{3/4}, \bar{X} \to -\infty$$
$$\bar{u}_0 = \bar{V}_0 = 0, \; \bar{Y}_3 = 0, \; \bar{X} < 0$$

and for the upper region *1* we obtain

$$\frac{\partial^2\bar{\varphi}_0}{\partial\bar{X}^2} + \frac{\partial^2\bar{\varphi}_0}{\partial\bar{Y}_1^2} = 0 \qquad (18)$$
$$\frac{\partial\bar{\varphi}_0}{\partial\bar{X}} = \bar{p}_0(\bar{X}), \quad \frac{\partial\bar{\varphi}_0}{\partial\bar{Y}_1} = -\frac{d\bar{A}_0}{d\bar{X}}, \quad \bar{X} < 0, \; \bar{Y}_1 = 0$$
$$\frac{\partial\bar{\varphi}_0}{\partial\bar{X}} = 0, \quad \bar{X} > 0, \quad \bar{Y}_1 = 0; \quad \bar{\varphi}_0 = \frac{2}{3}\,\mathrm{Re}\,[\mathrm{i}(\bar{X} + \mathrm{i}\bar{Y}_1)^{3/2}] + \ldots,$$
$$\bar{X}^2 + \bar{Y}_1^2 \to \infty; \quad \frac{\partial\bar{\varphi}_0}{\partial\bar{X}} = -\bar{P}_0(\bar{X}, \bar{Y}_1) = \bar{\rho}_0(\bar{X}, \bar{Y}_1)$$

The boundary value problems (17) and (18) were solved numerically in /8/. The transformation (16) can be regarded as a similarity law for subsonic flows of compressible fluids with a free streamline, past corner points.

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